

# DYNAMIC ANALYSIS OF LAMINATED COMPOSITE PLATES WITH HOLES

*A thesis  
submitted by*

**SanthoshPushpaRaj D  
(210CE2023)**

*In partial fulfillment of the requirements  
for the award of the degree of*

**Master of Technology  
In  
Civil Engineering  
(Structural Engineering)**

**Under The Guidance of  
Dr. ShishirKr.Sahu**



**Department of Civil Engineering  
National Institute of Technology Rourkela  
Orissa -769008, India  
May 2012**

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DEPARTMENT OF CIVIL ENGINEERING  
NATIONAL INSTITUTE OF TECHNOLOGY  
ROURKELA, ODISHA-769008

## CERTIFICATE

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This is to certify that the thesis entitled, “**DYNAMIC ANALYSIS OF LAMINATED COMPOSITE PLATES WITH HOLES**” submitted by **SANTHOSH PUSH PARAJ D** bearing roll no. **210ce2023** in partial fulfilment of the requirements for the award of **Master of Technology** degree in **Civil Engineering** with specialization in “**Structural Engineering**” during 2010-2012 session at the National Institute of Technology, Rourkela is an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date:

Place: Rourkela

**Prof. Shishir Kr. Sahu**

Department of civil Engineering

National Institute of technology

Rourkela, Odisha-769008

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Santhoshpushparaj D

## **ABSTRACT**

Fiber reinforced composites are finding increasing applications in the aerospace, marine, transportation, electrical, chemical, construction and consumer goods industries. In some of these applications the composites are subjected to dynamic loads. The composite structures may sometimes be provided with different types of holes for the purpose of assembling the components and units inside the structure, for passing the cables and control mechanisms, for inspection, maintenance and attachment to other units.

The effects of the variations of behaviour for different shape of holes by maintaining same length/height ratio and hole area ratio are studied.

Scope of this project is to find out the best location of the holes. The ANSYS software is used for analyzing the plates under different boundary conditions and different orientation of laminate. Eight-noded Shell99 is used throughout the analysis which is a linear element. Two different boundary conditions are considered those are CFFF-(clamped free free free) and CFCF-(clamped free clamped free) conditions and length to height ratio considered are 50 and 200. The hole area ratio is maintained as constant throughout the analysis as 0.04. Two different layers of laminate is considered those are 4 no's and 8 no's having six different orientations each.

The influence of the thickness parameter is inherent at higher modes of vibration. In this report we study the dynamic behaviour of various laminates with different holes.

# TABLE OF CONTENTS

CERTIFICATE		i
ACKNOWLEDGEMENTS		ii
ABSTRACT		iii
LIST OF FIGURES		vi
LIST OF TABLES		viii
LIST OF ABBREVIATIONS		ix
Chapter	Topic Name	Page No
Chapter 1	INTRODUCTION	
	1.1 Introduction	1
	1.2 Importance of present study	2
Chapter 2	REVIEW OF LITERATURE	3
Chapter 3	THEORY AND FORMULATIONS	
	3.1 Laminated composite plates	5
	3.2 Finite Element Formulation	9
	3.3 Shell Element	9
	3.4 Stiffness Matrix	11
	3.5 Element Mass Matrix	12

Chapter 4	MODELLING	
4.1	Modeling in ANSYS	13
4.2	Material properties	15
Chapter 5	RESULTS AND DISCUSSIONS	
5.1	Introduction	24
5.2	Boundary conditions	25
5.3	Convergence study	26
5.3	Validation	27
5.4	Results	28
Chapter 6	CONCLUSIONS	
6.1	Introduction	41
6.2	Further Scope	42
REFERENCES		43-46

## **LIST OF FIGURES**

Fig 1:	Laminated plate with hole
Fig 2:	Orthotropic layer with a circular hole
Fig 3:	Cross section of Laminate composite
Fig 4:	Eight noded isoparametric element
Fig 5:	Eight noded Shell99 Element in ANSYS
Fig 6:	Laminate plate of CFFF condition
Fig 7:	Laminate Plate with hole at centre of CFFF condition
Fig 8:	Laminate Plate with hole at both edges of CFFF condition
Fig 9:	Laminate Plate with hole at edge of CFFF condition
Fig 10:	Laminate Plate with hole at support of CFFF condition
Fig 11:	Laminate Plate with hole at middle support of CFFF condition
Fig 12:	Laminate Plate with CFCF condition
Fig 13:	Laminate Plate with hole at centre of CFCF condition
Fig 14:	Laminate Plate with hole at edge of CFCF condition
Fig 15:	Laminate Plate with hole at both edges of CFCF condition
Fig 16:	Laminate Plate with hole at centre of support of CFCF condition
Fig 17:	Laminate Plate with hole at edge of support of CFCF condition
Fig 18:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFFF for $a/h=50$
Fig 19:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFCF for $a/h=50$
Fig 20:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite



	plate with CFFF for $a/h=50$
Fig 21:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $a/h=50$
Fig 22:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFFF for $a/h=200$
Fig 23:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CF CF for $a/h=200$
Fig 24:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $a/h=200$
Fig 25:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $a/h=200$

## **LIST OF TABLES**

Table 1:	Showing convergence results for various mesh divisions
Table 2:	Showing validation of results for CFFF and CFCF conditions
Table 3:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFFF for $a/h=50$
Table 4:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFCF for $a/h=50$
Table 5:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $a/h=50$
Table 6:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFCF for $a/h=50$
Table 7:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFFF for $a/h=200$
Table 8:	Effect of fibre orientations for frequencies having 4 Layers of Laminated Composite plate with CFCF for $a/h=200$
Table 9:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $A/h=200$
Table10:	Effect of fibre orientations for frequencies having 8 Layers of Laminated Composite plate with CFFF for $a/h=200$

## **LIST OF ABBREVIATIONS**

The principal symbols used in this thesis are presented for easy reference.

English	
$A_{ij}, B_{ij}, D_{ij}$	Extensional, bending-stretching coupling, Bending shear stiffnesses
$a/h$	length to thickness ratio of the Lamina
$[B]$	Strain displacement matrix for the element
$[D]$	Stress-strain
$dx, dy$	Element length in x and y-direction
$E_1, E_2$	Young's moduli of a lamina along and across the fibers, respectively
$G_{12}, G_{13}, G_{23}$	Shear moduli of a lamina with respect to 1, 2 and 3 axes
$[K_e]$	The elastic stiffness matrix
$K_x, K_y, K_{xy}$	Curvatures of the plate
$[N]$	The shape function matrix
$N_i$	Shape function at a node i
$N_x, N_y, N_{xy}$	In-plane internal force resultants per unit length
$\{P_e\}$	The element load vector due to external transverse static load
$\{P_e^N\}$	The element load vector due to hygrothermal forces

	and moments
$Q_x, Q_y$	Transverse shear resultants.
$t$	Thickness of the plate
$u, v$	Displacements of the mid-plane along x and y axes, respectively
$u_i, v_i, w_i$	Displacements of node i along x, y and z axes, respectively
$w$	Displacement along z axis
$x, y, z$	System of co-ordinate axes
$Z_k, Z_{k-1}$	Bottom and top distance of lamina from mid-plane

## Greek

$\epsilon_x, \epsilon_y, \gamma_{xy}$	In-plane strains of the mid-plane.
$\theta$	Fiber orientation in a lamina
$\theta_x, \theta_y$	Rotations of the plate about x and y axes
$\nu_{12}, \nu_{21}$	Poisson's ratios
$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$	Partial derivatives with respect to x and y
$\eta, \xi$	Local natural co-ordinates of an element
$\phi$	Angle of twist of the twisted panel

## Mathematical Operators

$[ ]^{-1}$	Inverse of the matrix
$[ ]^T$	Transpose of the matrix

# 1. INTRODUCTION

## 1.1 INTRODUCTION

Composite materials constitute a group of materials formed by putting together at least two different materials. A reinforced concrete beam and a car tire are examples of such materials. The aim of this three-dimensional composition is to acquire a property which none of the constituents possesses: In other words, the target is to produce a material that possesses higher performance properties for a particular purpose than its constituent parts. Some of these properties are mechanical strength, corrosion resistance, high temperature resistance, heat conductivity, stiffness, lightness, and appearance. In accordance with this definition, there are several conditions that must be satisfied by the composite material. It must be man-made and not natural. It must comprise at least two different materials with different chemical components separated by distinct interfaces. Different materials must be put together in a three-dimensional unity. It must possess properties which none of the constituents possesses alone and that must be the aim of its construction.

The material must behave as a whole, *e.g.* the fiber and the matrix material (material surrounding the fibers) must be perfectly bonded (Classical Lamination Theory-CLT). Lamination is used to combine the best aspect of the constituent layers and bonding material in order to achieve a more useful material. The properties that can be emphasized by lamination are strength, stiffness, low weight, corrosion resistance, thermal insulation, *etc.* Laminates, as with many other structures, could have holes to serve various purposes. An obvious purpose is to accommodate a bolt.

Considering two different materials that are reinforcing bars (fibers) and surrounding materials (matrix material), mechanical properties of each layer are given in two directions.

## 1.2 IMPORTANCE OF PRESENT STUDY

Fiber reinforced composites are finding increasing applications in the aerospace, marine, transportation, electrical, chemical, construction and consumer goods industries. In some of these applications the composites are subjected to dynamic loads. The composite structures may sometimes be provided with different types of holes for the purpose of assembling the components and units inside the structure, for passing the cables and control mechanisms, for inspection, maintenance and attachment to other units. The stresses and deformations of steep gradient are induced around these cutouts. The influence of the thickness parameter is inherent at higher modes of vibration. In this paper we study the dynamic behavior of laminates with different holes.

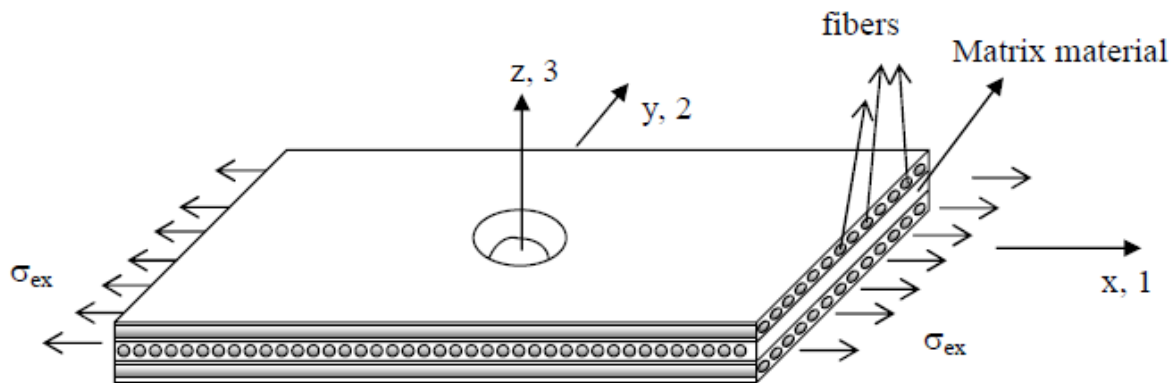


Figure-1 Laminated plate with hole

## 2.REVIEW OF LITERATURE

Akbarov, Yahnioglu and Babuscu Yesil [2] studied on the forced vibration initially statically stressed rectangular plate made of an orthotropic plate. plate is simply supported on all sides and a rectangular hole is present on edges. dynamic analysis are solve using three dimensional finite element method

Bailer, Hicks [5] developed theoretical method for determining elastic behavior of end loaded plates completely perforated with closely spaced circular holes. Method of solution was verified by experimental work. The following problems are considered in detail: (1) Unequal uniform applied extensions in the  $x$  and  $y$  directions; (2) Uniform applied shear. Using a digital computer, complete solutions have been obtained for the stress distribution in plates with holes.

Stahl. Keer [36] analysed eigenvalue problems of cracked rectangular plates. Vibration and buckling problems are solved for a plate with a crack emanating from one edge and for a plate with a centrally located internal crack. The problems are formulated as dual series equations and reduced to homogeneous Fredholm integral equations of the second kind. The singularity of the solution in each case is isolated and treated analytically. Numerical results for the natural frequencies and moment distributions are compared with the work of other investigators. Vibration and buckling mode shapes are also illustrated for a cracked plate.

Jwalamalini, Sundaravadivelu, Vendhan, Ganapathy[19] The stability of a simply supported square plate with openings under in-plane loading is analyzed using a Finite Element program BUCSAP (Buckling Structural Analysis Program). The openings are considered as square and central for the main study but rectangular and central for comparison with other work. Different magnitudes of tension and compression are assumed as initial pre-stress in the transverse direction before the longitudinal stress is applied.

Cheng [12] the formal solution of the problem of defraction of a plane, time-harmonic, compressional wave by a group of cavities in a thin elastic plate is obtained by the method of multiple scattering. The cavities are circular and their geometry of distribution is arbitrary. Numerical results of two identical holes at a finite separation are presented in detail.

Myung Jo Jung, Young Hwan Choi and Yong ho Ryu [28] Free vibration analysis of circular plate with eccentric hole submerged in fluid studied the natural frequencies and mode shapes of the structures due to the existence of hole. Especially if the hole is located eccentrically the vibration behavior of the structures is expected to deviate significantly from that of a plate with concentric hole.

Liew, ng and Kitipornchai [25] A semi analytical analysis of free vibration of plates with discontinuities in cross section and changes in thickness are considered. A square element is used a basic building element .continuities in displacement, slope, moment and higher derivatives between adjacent subdomains are enforced at the interconnecting edges. Ritz procedure is used to extract the frequencies and mode shapes.

Bicos, George, Springer [6] Equations are derived which describe the free damped vibrations of plates and shells made of laminated fiber-reinforced, organic-matrix composites. A finite element method is developed for obtaining solutions to these equations. A computer code is written, which can be used to calculate the natural frequencies, mode shapes, and damping factors of rectangular plates, cylinders, and cylindrical panels with free, clamped, or simply supported edges, and with or without circular cutouts. Natural frequencies and mode shapes calculated by the code for isotropic and composite plates, cylinders, and cylindrical panels are compared with previous analytical, numerical, and experimental results. The results of the present study agree closely with those reported by previous investigators.



### 3.THEORY AND FORMULATION

#### ANALYSIS

##### 3.1 LAMINATED COMPOSITE PLATES

The stress-strain relation for a three-dimensional linear, elastic, anisotropic material is given as

$$\{\sigma\} = [C]\{\varepsilon\}$$

which is also known as Hooke's law.  $\{\sigma\}$  and  $\{\varepsilon\}$  are stress and strain vectors respectively. The  $[C]$  matrix is called material stiffness matrix, which has 21 independent material constants. For plane stress problems, where the external stresses are in the plane of plate, Hooke's law could be simplified to

$$\underbrace{\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}}_{\{\sigma\}} = \underbrace{\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}}_{[Q]} \underbrace{\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}}_{\{\varepsilon\}}$$

in which  $[Q]$  is the reduced material stiffness matrix, having elements as

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} \\ Q_{12} &= Q_{21} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{66} &= G_{12} \end{aligned}$$

Here,  $E_1$  is the elasticity modulus in the fiber direction,  $E_2$  is the elasticity modulus in the transverse direction,  $\nu_{12}$  and  $\nu_{21}$  are the Poisson's ratio, and  $G_{12}$  is the shear modulus. For a unidirectional fiber reinforced layer there are two principal material directions. One corresponds to fiber direction and the other corresponds to matrix material direction denoted by subscripts 1

and 2, respectively. When these material directions are oriented by the angle  $\alpha$  from the plate direction (Figure 2), the stress strain relation is given as

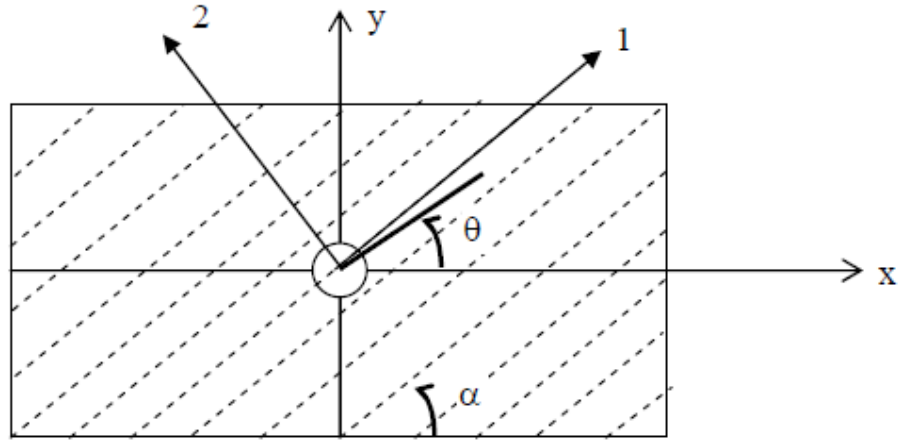


Figure 2. Orthotropic layer with a circular hole

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$[\bar{Q}] = [T]^{-1} [Q] [R] [T] [R]^{-1}$$

$$[T] = \begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha & 2 \sin \alpha \cos \alpha \\ \sin^2 \alpha & \cos^2 \alpha & -2 \sin \alpha \cos \alpha \\ -\sin \alpha \cos \alpha & \sin \alpha \cos \alpha & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix}$$

$$[R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Classical Lamination Theory (CLT) assumes that all layers are perfectly bonded together in a plate and the in plane deformations are continuous. In case of loading, the strain distribution could be rewritten as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

The first vector on the right hand side is mid-plane strains, the second is curvatures, and  $z$  is depth from mid plane. For a laminated composite plate (Figure 3), the relation between applied forces and plate strains is given as

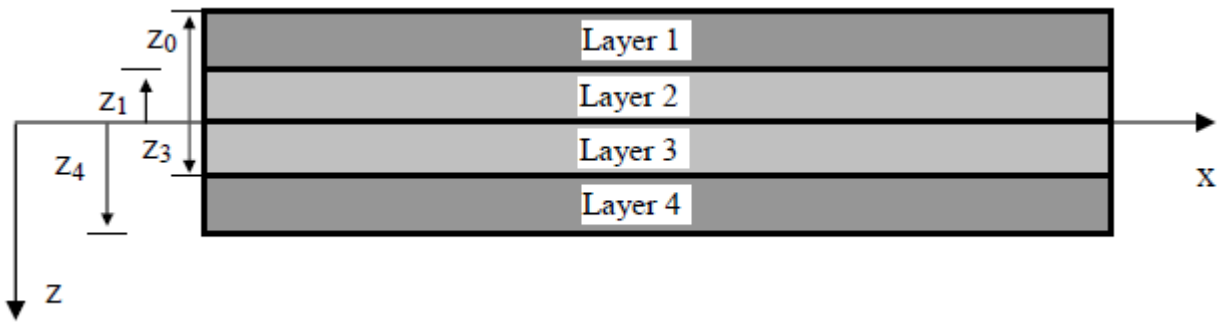


Figure 3- crossection of laminated composite

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \end{Bmatrix}$$

where elements of  $[A]$ ,  $[B]$ , and  $[D]$  matrices are defined as

$$A_{ij} = \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

in which  $n$  is the “number of layers.” For a given loading value of the laminated plate, Equation (7) is solved for plate strains and curvatures. To obtain the layer stress state, mid-plane strains and curvatures, which are the same for all layers, are put in Equation (4) and solved for stresses. It is obvious that due to symmetric lamination, no moments are calculated.

The governing differential equations, the strain energy due to loads, kinetic energy and formulation of the general dynamic problem are derived on the basis of the principle of potential energy and Lagrange’s equation.

$$[Q_{ij}]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad \text{For } i,j = 1,2,6$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \quad \text{For } i,j = 4,5$$

$$Q_{11} = \frac{E_1}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{12} = \frac{\vartheta_{12}E_1}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{22} = \frac{E_2}{(1 - \vartheta_{12}\vartheta_{21})}$$

$$Q_{44} = G_{13}, Q_{55} = G_{23}$$

$E_1, E_2$  = Young’s moduli of a lamina along and across the fibers, respectively

$G_{12}, G_{13}, G_{23}$  = Shear moduli of a lamina with respect to 1, 2 and 3 axes.

$\vartheta_{12}, \vartheta_{21}$  = Poisson’s ratios

### 3.2FINITE ELEMENT FORMULATION

For problems involving complex geometrical and boundary conditions, analytical methods are not easily adaptable and numerical methods like finite element methods (FEM) are preferred. The finite element formulation is developed hereby for the structural analysis of isotropic as well as composite twisted panels using a curved shear deformable shell theory.

### 3.3 SHELL ELEMENT

The plate is made up of perfectly bonded layers. Each lamina is considered to be homogeneous and orthotropic and made of unidirectional fiber-reinforced material. The orthotropic axes of symmetry in each lamina are oriented at an arbitrary angle to the plate axes. An eight-noded isoparametric quadratic shell element is employed in the present analysis with five degrees of freedom  $u$ ,  $v$ ,  $w$ ,  $\theta_x$  and  $\theta_y$  per node as shown in Figure. But the in-plane deformations  $u$  and  $v$  are considered for the initial plane stress analysis. The isoparametric element shall be oriented in the natural coordinate system and shall be transferred to the Cartesian coordinate system using the Jacobian matrix. In the analysis of thin shells, where the element is assumed to have mid-surface nodes, the shape function of the element is derived using the interpolation polynomial.

For problems involving complex in-plane loading and boundary conditions numerical methods like finite element method (FEM) are preferred. Eight-noded isoperimetric element is used to the present free vibration problem. Five degrees of freedom  $u$ ,  $v$ ,  $w$ ,  $\theta_x$  and  $\theta_y$  are considered at each node. The stiffness matrix, the geometric stiffness matrix due to residual stresses, geometric stiffness matrix due to applied in-plane loads and nodal load vector of the element are derived using the principle of minimum potential energy.

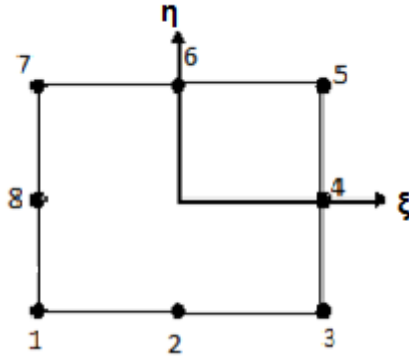


Figure 4 Eight noded isoparametric element

The element displacements are expressed in terms of their nodal values by using the element shape functions and are given by

$$u = \sum_{i=1}^8 N_i v_i \quad v = \sum_{i=1}^8 N_i v_i \quad w = \sum_{i=1}^8 N_i w_i$$

$$\theta_x = \sum_{i=1}^8 N_i \theta_{xi} \quad \theta_y = \sum_{i=1}^8 N_i \theta_{yi}$$

$N_i$  = Shape function at a node  $i$

$\xi, \eta$  = Local natural co-ordinates of an element

### 3.4 STIFFNESS MATRIX

The linear strain matrix  $\{\epsilon\}$  is obtained by substituting equations (11) into (9), and is expressed as

$$\{\varepsilon\} = [B]\{\delta e\} \dots\dots\dots$$

Where

$$\{\delta e\} = \{u_1 \ v_1 \ w_1 \ \theta_{x1} \ \theta_{y1} \ \dots \ \dots \ \dots \ \dots \ \dots \ \dots \ u_8 \ v_8 \ w_8 \ \theta_{x8} \ \theta_{y8}\}^T$$

$$[B] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & \frac{N_i}{R_x} & 0 & 0 \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 2 \frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial N_i}{\partial y} \\ 0 & 0 & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \\ 0 & 0 & \frac{\partial N_i}{\partial x} & N_i & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & N_i \end{bmatrix}$$

The elastic stiffness matrix is given by

$$[K_e] = \iint [B]^T [D] [B] dx dy$$

### 3.5 ELEMENT MASS MATRIX

$$[M_e] = \iint [N]^T [P] [N] dx dy$$

Where the shape function matrix

$$[N] = \sum_{i=1}^8 \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix}$$

$$[P] = \begin{bmatrix} P_1 & 0 & 0 & P_2 & 0 \\ 0 & P_1 & 0 & 0 & P_2 \\ 0 & 0 & P_1 & 0 & 0 \\ P_2 & 0 & 0 & P_3 & 0 \\ 0 & P_2 & 0 & 0 & P_3 \end{bmatrix}$$

In which

$$(P_1 P_2 P_3) = \sum_{k=1}^n \int_{z_{k-1}}^{z_k} (\rho)_k (1, z, z^2) dz$$

The element load vector due to external transverse static load q per unit area is given by

$$\{P_e\} = \iint N_i \begin{bmatrix} q \\ 0 \\ 0 \end{bmatrix} dx dy$$

The element load vector due to hygrothermal forces and moments is given by

$$\{P_e^N\} = \iint [B]^T \{F^N\} dx dy$$



## **4. MODELLING**

### **4.1 MODELLING IN ANSYS**

ANSYS is finite element based software which gives good results on analysis of any structural elements. It has the capability to analyze multi layer laminated composite with different orientation. SHELL99 is used as an modeling element

#### **SHELL99**

It may be used for layered applications of a structural shell model. SHELL99 is having only the linear capability; it usually has a smaller element formulation time. It allows up to 250 layers. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes

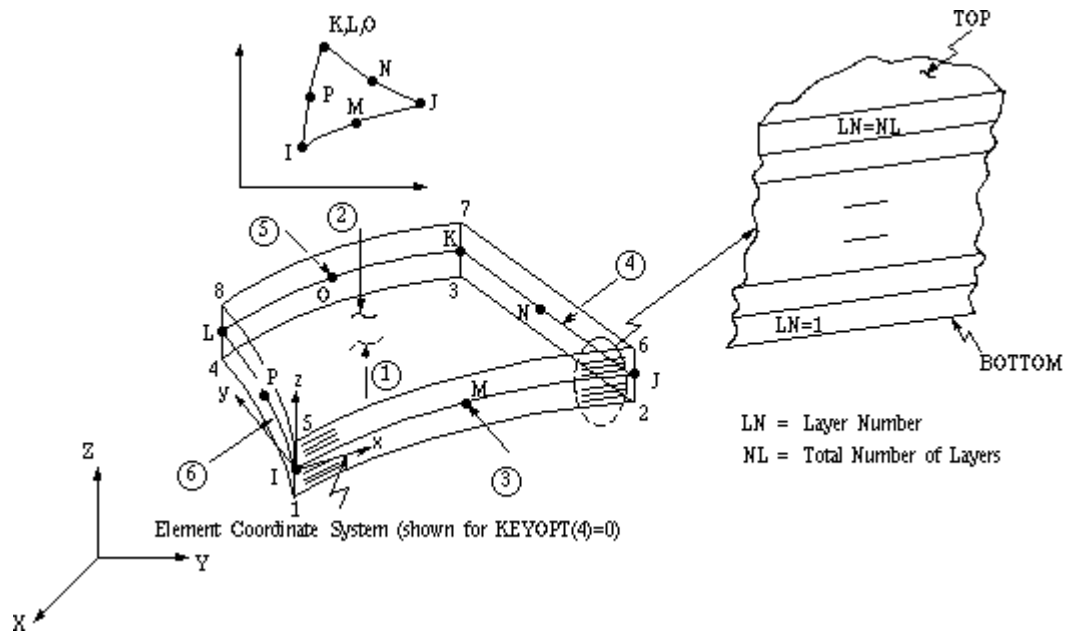


Figure 5- Eight noded shell99 element in ANSYS

The element is defined by eight nodes, average or corner layer thicknesses, layer material direction angles, and orthotropic material properties. Midsize nodes may not be removed from this element. The material properties of each layer may be orthotropic in the plane of the element. Throughout the analysis ratio of length to thickness is kept constant as 125.

The two different support conditions are taken

1. CFFF-Clamped on one edge and free on remaining edges
2. CFCF-Clamped on opposite edges and free on other opposite edges

## 4.2 MATERIAL PROPERTIES

Material properties considered throughout the analysis are

$E_{11}=134.4\text{Gpa}$ ,  $E_{22}=10.34\text{Gpa}$ ,  $\nu_{12}=0.33$ ,  $\nu_{21}=0.33$ ,  $G_{12}=4.99\text{Gpa}$ ,  $G_{23}=1.999\text{Gpa}$ ,  $G_{13}=4.99\text{Gpa}$ . [2]

Where  $E_{11}$  is young's modulus in 1-1 axis,  $E_{22}$  is young's modulus in 2-2 axis.  $\nu_{12}$  is poisons ratio in 1-2 axis,  $\nu_{21}$  is poisons ration in 2-1 axis and  $G_{12}, G_{23}, G_{13}$  are shear stress respectively.

Analysing of plate with shell element shell99 with hole area to total area as ( $a/A=0.04$ ) and total area to total thickness as ( $A/h=50$ ) the total thickness will be 0.005m and ( $A/h=200$ ). For the ( $A/h=200$ ) the total thickness will be 0.00125m. Figure no? shows the model in ansys having different location of holes

### **MODEL –I**

The figure shows the model-I(CFFF) which is an four layer laminate of orientation as 0/90/90/0 with each layer thickness as 0.00125 considering different locations of holes as I(a) No holes I(b)Hole at centre I(c)Hole at edge,I(d)Hole at both edges,I(e)Hole at support (mid),I(f)Hole at support(corner).

### **MODEL –II**

The figure shows the model-II (CFFF) which is a four layer laminate of orientation as 0/45/45/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL –III**

The figure shows the model-III (CFFF) which is an four layer laminate of orientation as 0/60/60/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-IV**

The figure shows the model-IV (CFFF) which is an four layer laminate of orientation as 0/30/30/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-V**

The model-V(CFFF) which is a four layer laminate of orientation as 0/30/60/90 with each layer thickness as 0.00125 considering all the above conditions.

**MODEL-VI** The model-VI(CFFF) which is a four layer laminate of orientation as 0/30/60/90 with each layer thickness as 0.00125 considering all the above conditions.

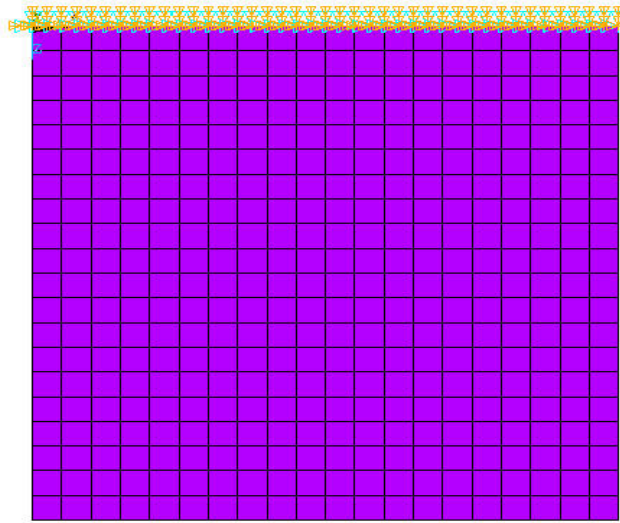


Figure 6 - laminate plate of CFFF condition

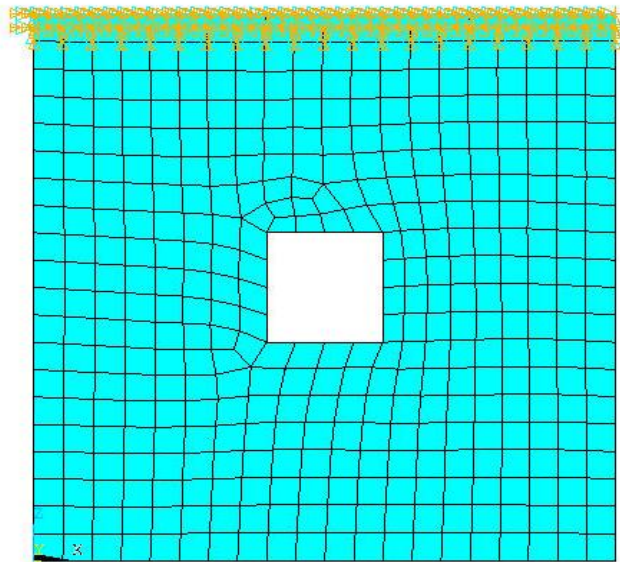


Figure 7- Laminate plate with hole at centre of CFFF condition

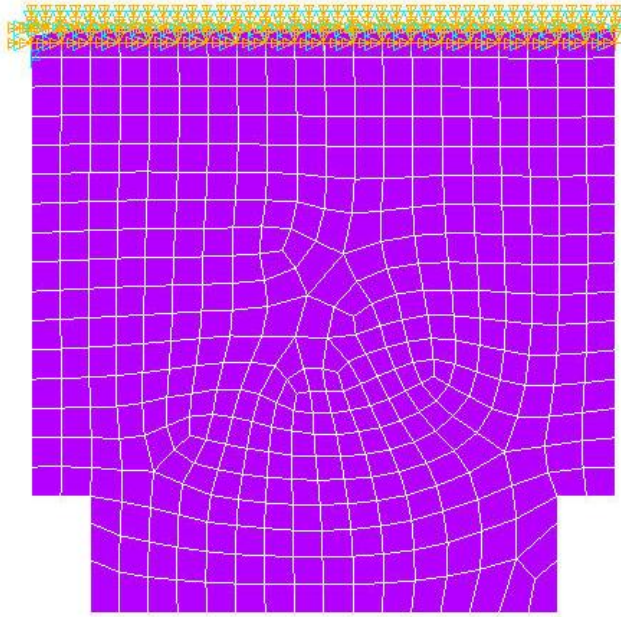


Figure 8- Laminate plate with hole at both edges of CFFF condition

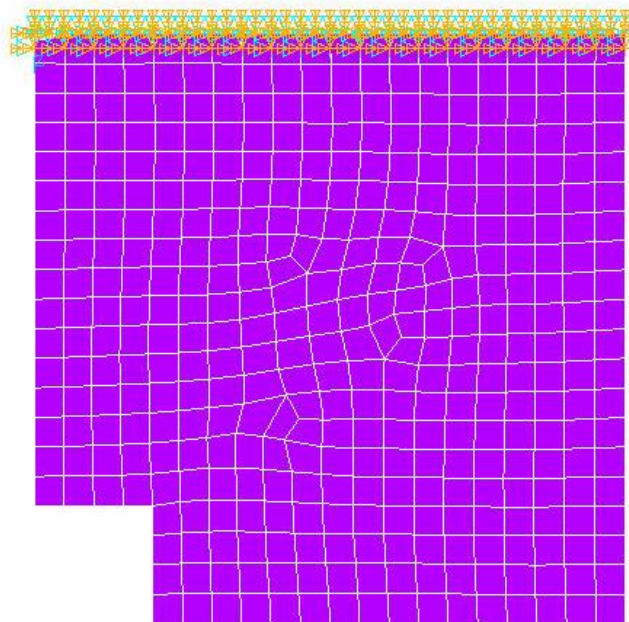


Figure 9- Laminate plate with hole at edge with CFFF condition

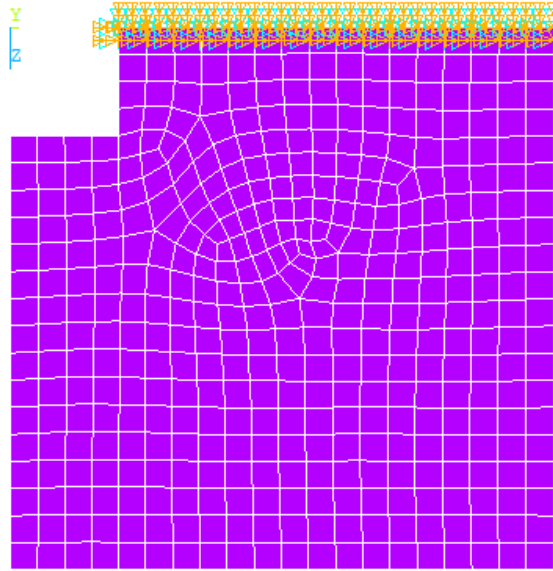


Figure 10- Laminate plate with Hole at support of CFFF condition

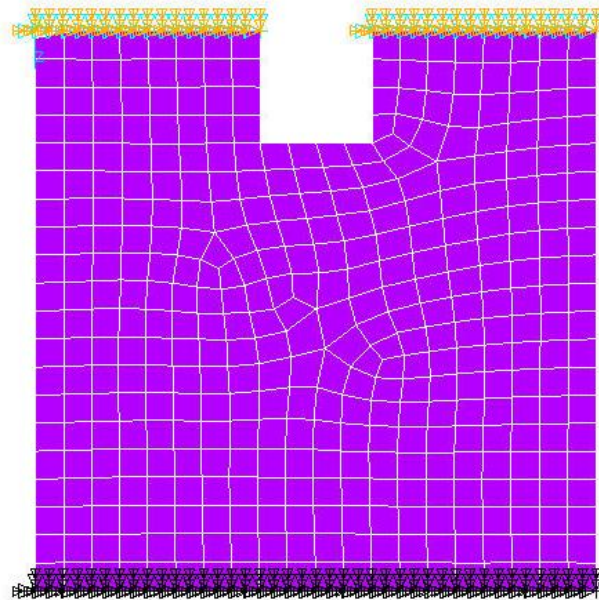


Figure 11- Laminate plate with hole at middle support of CFFF condition

## **CFCF CONDITION**

### **MODEL-VII**

The model-VII (CFCF) which is a four layer laminate of orientation as 0/90/90/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-VIII**

The model-VIII (CFCF) which is a four layer laminate of orientation as 0/45/45/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-IX**

The model-IX (CFCF) which is a four layer laminate of orientation as 0/60/60/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-X**

The model-X (CFCF) which is a four layer laminate of orientation as 0/30/30/0 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-XI**

The model-XI (CFCF) which is a four layer laminate of orientation as 0/30/60/90 with each layer thickness as 0.00125 considering all the above conditions.

### **MODEL-XI**

The model-XII (CFCF) which is a four layer laminate of orientation as 0/15/30/45 with each layer thickness as 0.00125 considering all the above conditions and for 8Layers.



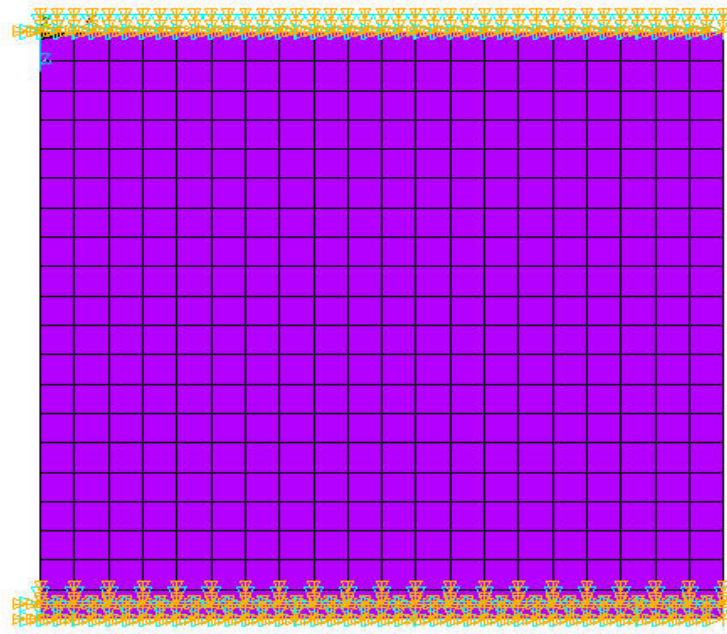


Figure 12- Laminated plate with CFCF condition

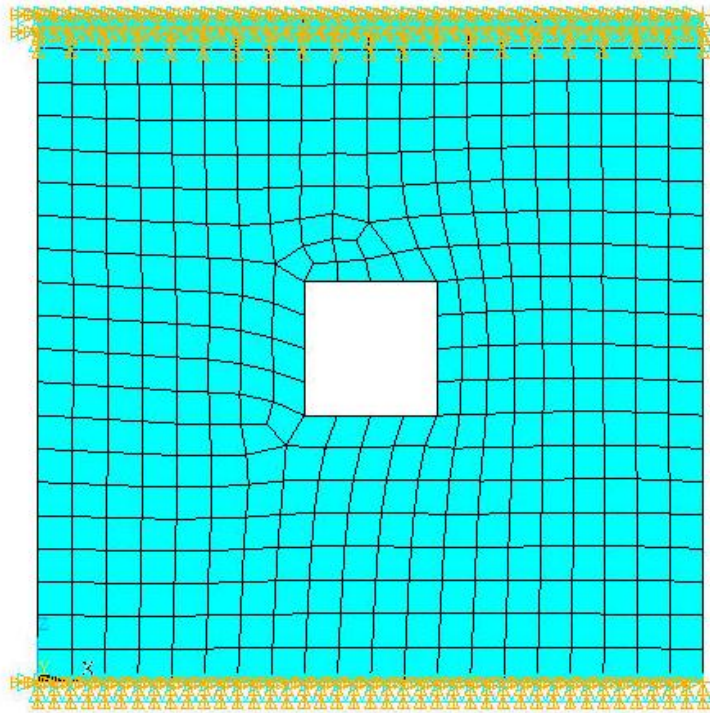


Figure 13- Laminate plate with hole at centre of CFCF condition

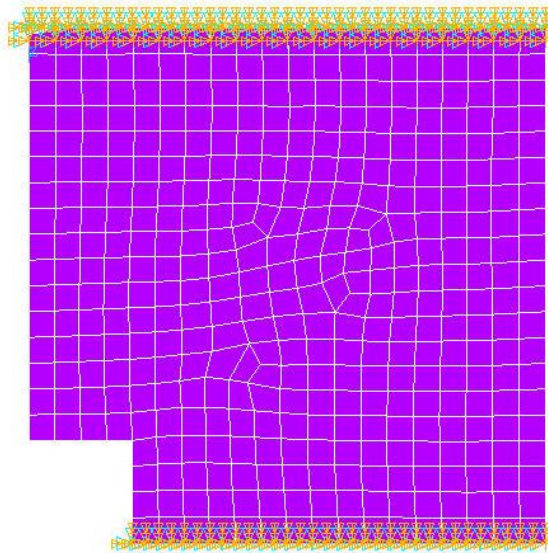


Figure 14- Laminate plate with hole at edge of CFCF condition

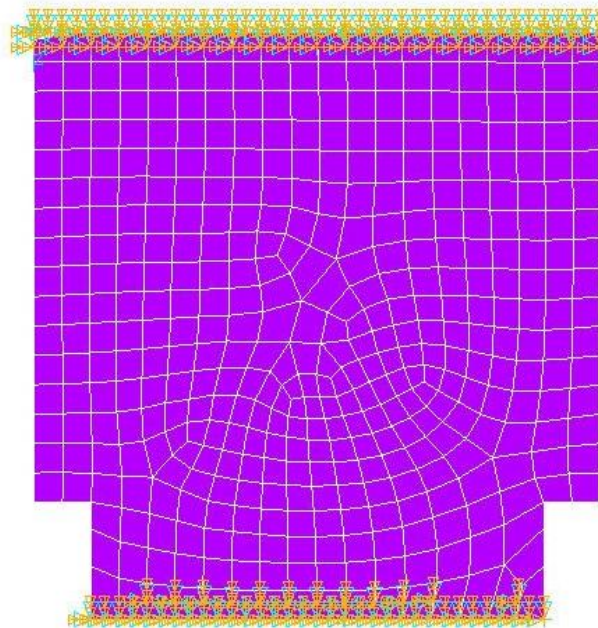


Figure 15- Laminate plate with hole at both edges of CFCF condition

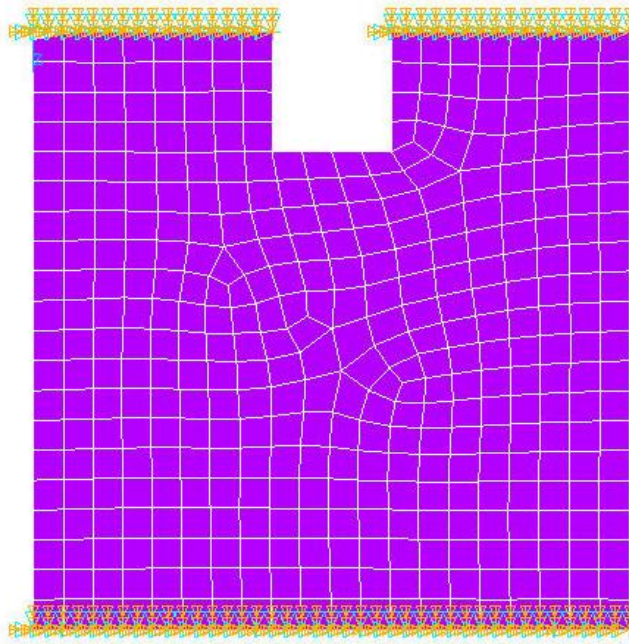


Figure 16- Laminate plate with hole at centre of support of CFCF condition

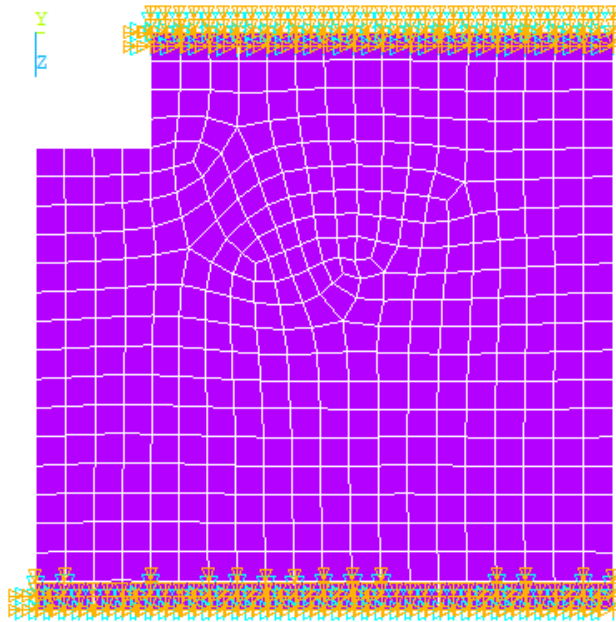


Figure 17- Laminate plate with hole at edge of support of CFCF condition.

## **5. RESULTS AND DISCUSSIONS**

### **5.1 INTRODUCTION**

The composites plates with arbitrary geometries and boundary conditions subjected to various loading got important roles to play as the structural elements in aerospace and other engineering structures. The plate and shell structures subjected to dynamic loading cause non-uniform stress field which greatly affects the stability and dynamic behavior of structures.

Here the results of free vibration are plotted with various different angles of laminate orientations. The laminated composites considered for the present analysis are clamped free free free condition (CFFF) and Clamped free clamped free conditions (CFCF).

## 5.2 BOUNDARY CONDITIONS

Numerical results are presented for laminated composite plates with different combinations of boundary conditions. Plates of same geometry having size as 0.5 by 0.5 m. having a hole area ratio as 0.04. In the discussion that followed F and C denote No Support, clamped Edges respectively. For comparison problems, the boundary conditions are considered as reported in the respective studies.

Two types of boundary conditions are described as follows:

### (i) Cantilever

CFFF:  $u=v=w=\theta_x=\theta_y=0$  at  $x=0$ ,

### (ii) Clamped boundary

CCCC:  $u=v=w=\theta_x=\theta_y=0$  at  $x=0, a$  and  $y=0, b$

The results are presented for symmetric cross-ply and anti-symmetric angle-ply laminated composite plates are considered.

### 5.3 Convergence study

The convergence study is done for non-dimensional frequencies of free vibration of

CFFF square 4 layer symmetric cross ply laminated composite plates for different mesh division as shown in Table 2,. Sufficient number of convergence tests are made and found that 0.025 is used as an edge element length in ANSYS and which gives total number of elements used in project is 400 elements. The study is further extended to effect of dynamic analysis of laminated composite plates with holes.

Convergence of free vibration frequencies for CFFF 4 layer plates for different ply orientations.

TABLE NO.1 SHOWING CONVERGENCE RESULTS FOR VARIOUS MESH DIVISIONS

Mesh Division	Frequency ( $\omega$ ), Hertz	
	0/90/90/0	0/45/45/0
4 x 4	30.104	30.564
8 x 8	28.200	28.5704
10 x 10	27.932	28.298
20 x 20	27.932	28.298

### 5.3 VALIDATION

Comparison of free vibration frequencies for CFFF and CFCF (0/90)<sub>10</sub> square plate Carbon Fiber specimen with 4 mm thickness 150mm X 150mm size

$$E1 = 172.7 \text{ GPa}, E2 = E3 = 7.2 \text{ GPa}, G12 = G13 = G23 = 3.76 \text{ GPa}, V_{12} = V_{23} = V_{13} = 0.3$$

TABLE NO.2 SHOWING VALIDATION OF RESULTS FOR CFFF AND CFCF CONDITIONS

CFFF			CFCF		
mode no	Dutt et al.[26]	present	mode no	Dutt et al.[26]	present
1	37.98	37.74	1	152.96	152.47
2	47.65	47.44	2	223.76	223.36
3	63.65	63.53	3	345.98	345.48
4	89.12	88.54	4	435.29	434.78
5	103.86	103.45	5	587.76	587.25

## 5.4 RESULTS

The increase in frequency in any case is due to the increase in stiffness of the plate and/or due to the decrease in mass of the plate for any change in the geometry of the plate. The decrease in frequency at any position is due to the decrease in stiffness of the plate. In some of the modes it is observed that there is no significant variation in frequency.

Modal analysis is done having 4 layers of composite having CFFF condition with Six different orientations are considered, by keeping  $a/h$  ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/30/30/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 1 and the figure no. shows the plot of 4 layers of laminated composite with CFFF conditions. For all others cases (0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.

Table NO. 3    EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4  
LAYERS OF LAMINATED COMPOSITE WITH CFFF



	0/90/90/0	0/60/60/0	0/45/45/0	0/30/30/0	0/15/30/45	0/30/60/90
NO HOLES	27.932	28.019	28.298	28.804	17.015	13.779
HOLE AT CENTRE	18.871	17.243	16.582	16.206	14.233	16.052
HOLE AT EDGE	22.091	20.384	19.718	19.423	16.07	16.427
HOLE AT BOTH EDGES	22.957	21.974	21.465	21.244	16.729	16.883
HOLE AT SUPPORT	15.894	14.586	13.624	12.874	12.482	14.489
HOLE AT SUPPORT(CORNER)	18.681	17.84	17.535	17.313	13.919	13.138

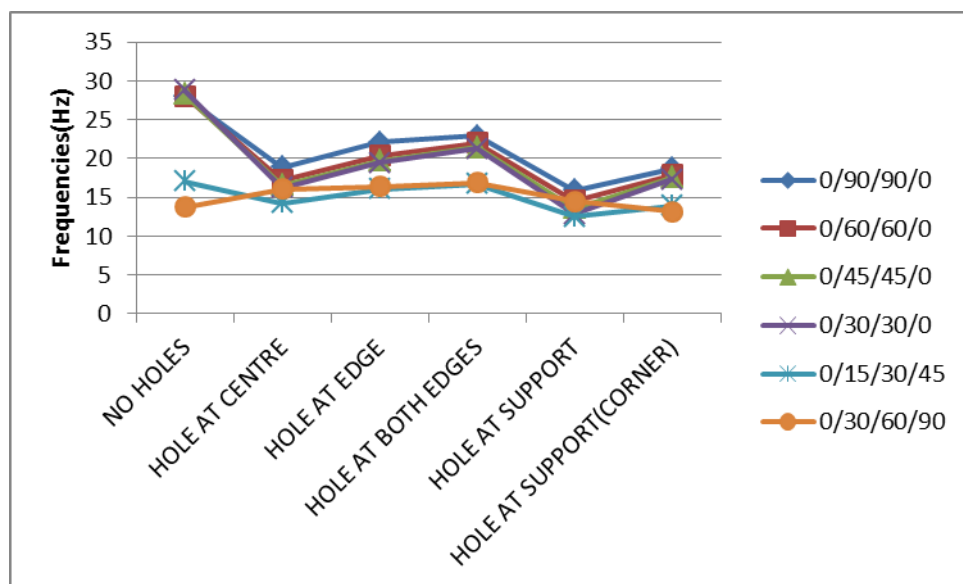


Figure 18- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS OF LAMINATED COMPOSITE WITH CFFF

Linear Dynamic analysis is done having 4 layers of composite having CFCF condition with Six different orientations are considered, by keeping a/h ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/30/30/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 2 and the figure no. shows the plot of 4 layers of laminated composite with CFCF conditions. For all others case s (0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.

Table NO. 4 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS OF LAMINATED COMPOSITE WITH CFCF

	0/90/90/0	0/60/60/0	0/45/45/0	0/30/30/0	0/15/30/45	0/30/60/90
NO HOLES	176.4	170.08	179.12	182.66	109.8	87.683
HOLE AT CENTRE	128.98	124.36	120.94	117.45	96.723	105.78
HOLE AT EDGE	110.98	110.43	107.7	104.39	93.953	93.238
HOLE AT BOTH EDGES	117.86	113.49	109.63	105.85	95.202	106.48
HOLE AT SUPPORT(MID)	122.27	116.34	111.66	108.41	92.174	102.58
HOLE AT SUPPORT(CORNER)	103.67	100.5	97.757	94.617	83.592	88.629

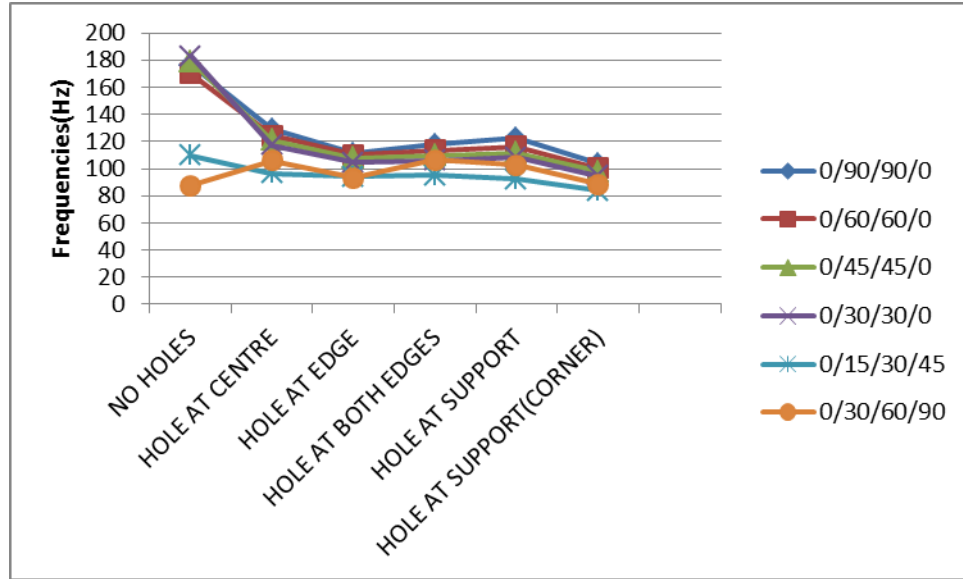


Figure 19- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS OF LAMINATED COMPOSITE WITH CFCF

Linear Dynamic analysis is done having 8 layers of composite having CFFF condition with Six different orientations are considered, by keeping  $a/h$  ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/15/30/45/45/30/15/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 5 and the figure no. shows the plot of 8 layers of laminated composite with CFFF conditions. For all others cases (0/90/90/0/0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.

Table NO. 5 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8  
LAYERS OF LAMINATED COMPOSITE WITH CFFF

	(0/90/90/0)s	(0/60/60/0)s	(0/45/45/0)s	(0/30/30/0)s	(0/15/30/45)s	(0/30/60/90)s
NO HOLES	23.486	23.797	24.671	26.254	27.592	25.465
HOLE AT CENTRE	21.868	18.83	17.479	16.615	16.485	18.125
HOLE AT EDGE	23.195	21.228	19.922	19.227	19.268	20.57
HOLE AT BOTH EDGES	24.239	21.57	20.628	20.431	20.742	21.406
HOLE AT SUPPORT(MID)	19.111	16.301	14.824	13.658	13.35	15.152
HOLE AT SUPPORT(CORNER)	19.126	18.371	18.333	18.112	17.825	18.757

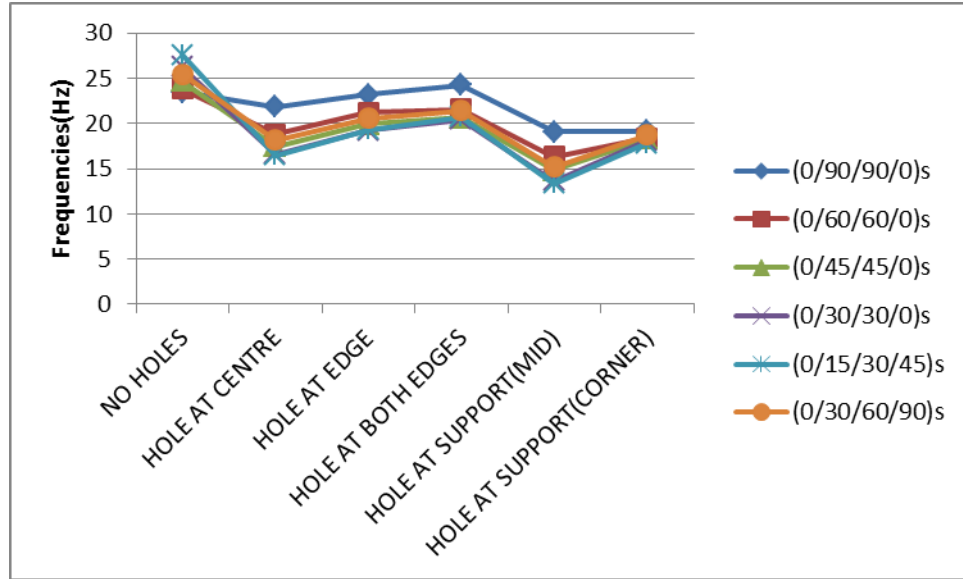


Figure 20 - EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8 LAYERS OF LAMINATED COMPOSITE WITH CFFF

Linear Dynamic analysis is done having 8 layers of composite having CFCF condition with Six different orientations are considered, by keeping a/h ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/15/30/45/45/30/15/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 4 and the figure no. shows the plot of 4 layers of laminated composite with CFCF conditions. For all others cases (0/90/90/0/0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.

Table NO. 6 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8  
LAYERS OF LAMINATED COMPOSITE WITH CFCF

	(0/90/90/0)s	(0/60/60/0)s	(0/45/45/0)s	(0/30/30/0)s	(0/15/30/45)s	(0/30/60/90)s
NO HOLES	148.65	150.94	157.67	169.51	177.58	163.25
HOLE AT CENTRE	144.17	130.35	123.37	120.26	119.34	126.41
HOLE AT EDGE	113.08	110.51	103.11	95.185	91.605	102.81
HOLE AT BOTH EDGES	133.21	123.14	116.37	111.53	108.24	118.45
HOLE AT SUPPORT(MID)	132.47	118.26	111.13	107.82	108.11	114.42
HOLE AT SUPPORT(CORNER)	115.8	102.68	97.186	92.357	90.243	98.833

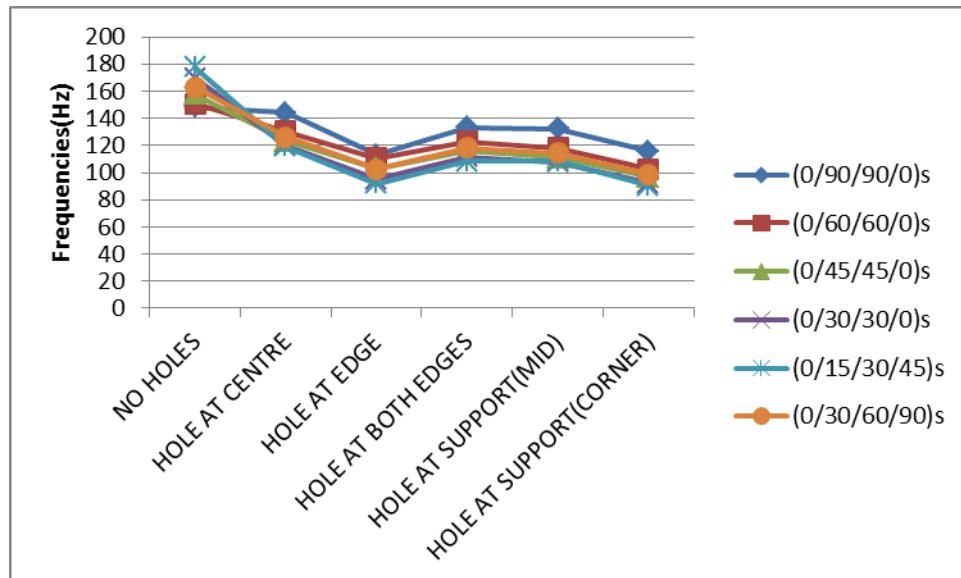


Figure 21- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8 LAYERS  
OF LAMINATED COMPOSITE WITH CFCF

### FOR A/h=200

Linear Dynamic analysis is done having 4 layers of composite having CFFF condition with Six different orientations are considered, by keeping a/h ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/30/60/90) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 5 and the figure no. shows the plot of 4 layers of laminated composite with CFFF conditions. For all others cases (0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.

Table NO. 7 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS OF LAMINATED COMPOSITE WITH CFFF

For a/h=200

	0/90/90/0	0/60/60/0	0/45/45/0	0/30/30/0	0/15/30/45	0/30/60/90
NO HOLES	3.4078	2.8244	2.4695	2.2539	2.6064	3.6148
HOLE AT CENTRE	4.8181	4.442	4.271	4.1669	3.5875	3.8445
HOLE AT EDGE	5.4051	5.1996	4.9254	4.7099	4.0326	4.1057
HOLE AT BOTH EDGES	5.7872	5.4519	5.2321	5.1	4.2861	4.463
HOLE AT SUPPORT	4.0116	3.6537	3.4125	3.2309	3.2152	3.7422
HOLE AT SUPPORT(CORNER)	4.557	4.3332	4.2418	4.1714	3.4775	3.3994

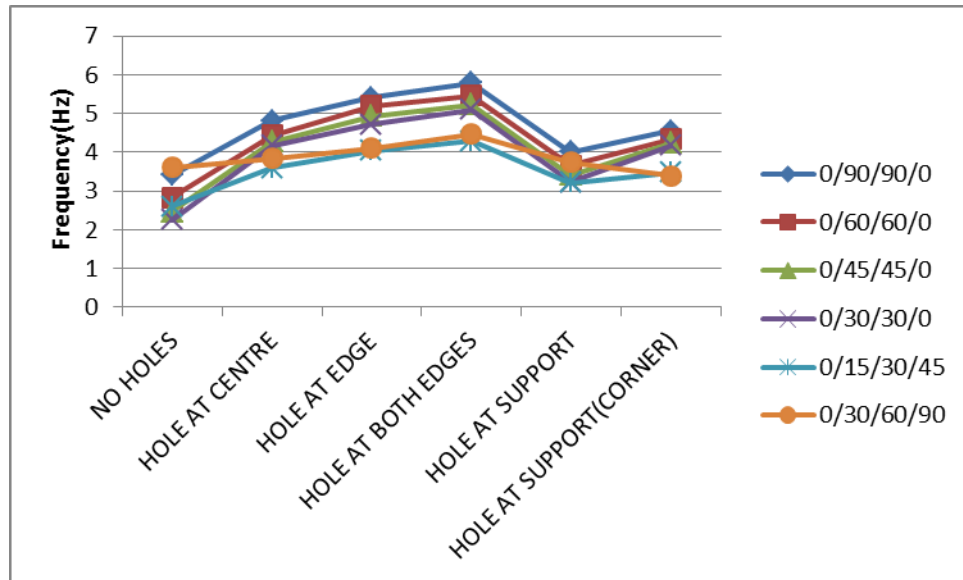


Figure 22- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS OF LAMINATED COMPOSITE WITH CFFF

Linear Dynamic analysis is done having 4 layers of composite having CFCF condition with Six different orientations are considered, by keeping  $a/h$  ratio as constant which is equal to 0.04. In case 1 (no holes in plate) (0/30/60/90) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 8 and the figure no. shows the plot of 4 layers of laminated composite with CFCF conditions. For all others cases (0/90/90/0) gives the good results as compared with other. But plate is affected less when Hole is at the center of the plate.



Table NO. 8 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4  
LAYERS OF LAMINATED COMPOSITE WITH CFCF

	0/90/90/0	0/60/60/0	0/45/45/0	0/30/30/0	0/15/30/45	0/30/60/90
NO HOLES	21.689	18.485	16.031	14.425	17.16	23.042
HOLE AT CENTRE	31.659	30.465	29.547	25.565	24.222	26.78
HOLE AT EDGE	25.533	25.222	24.408	23.389	21.829	22.493
HOLE AT BOTH EDGES	29.054	28.254	27.292	26.095	24.576	27.124
HOLE AT SUPPORT	30.752	29.052	27.835	27.074	23.468	26.37
HOLE AT SUPPORT(CORNER)	25.235	23.768	22.978	22.27	19.875	21.556

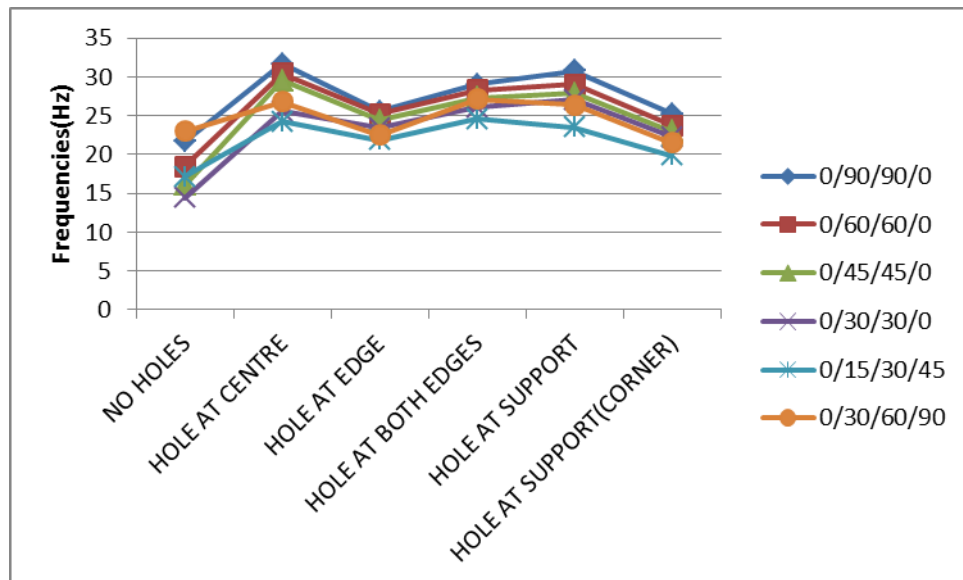


Figure 23- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 4 LAYERS  
OF LAMINATED COMPOSITE WITH CFCF

Linear Dynamic analysis is done having 8 layers of composite having CFFF condition with Six different orientations are considered, by keeping a/h ratio as constant which is equal to 0.04. In the case of (0/90/90/0/0/90/90/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 7 and the figure no. shows the plot of 4 layers of laminated composite with CFCF conditions. But plate is affected less when Hole is at the center of the plate.

Table NO. 9 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8 LAYERS OF LAMINATED COMPOSITE WITH CFFF

	(0/90/90/0)s	(0/60/60/0)s	(0/45/45/0)s	(0/30/30/0)s	(0/15/30/45)s	(0/30/60/90)s
NO HOLES	5.2015	3.645	2.8541	2.3809	2.2956	2.9978
HOLE AT CENTRE	5.3183	4.5577	4.2944	4.1713	4.1763	4.4583
HOLE AT EDGE	5.7973	5.3267	4.9663	4.7241	4.6984	5.112
HOLE AT BOTH EDGES	6.1027	5.4872	5.1878	5.0354	5.0659	5.3662
HOLE AT SUPPORT	4.8869	4.1254	3.6983	3.3638	3.2743	3.7761
HOLE AT SUPPORT(CORNER)	4.6781	4.3851	4.3336	4.2827	4.2533	4.4653

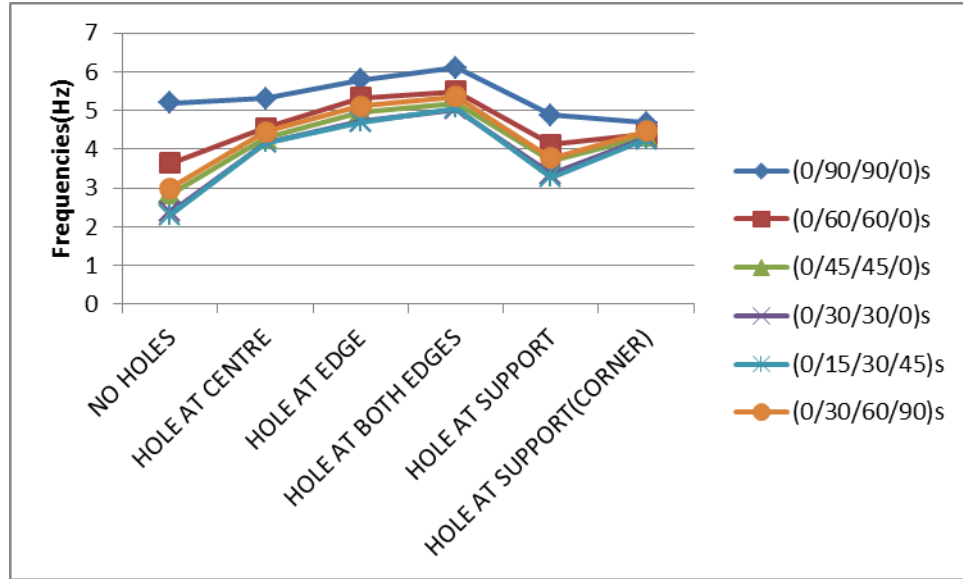


Figure 24- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8 LAYERS OF LAMINATED COMPOSITE WITH CFFF

Linear Dynamic analysis is done having 8 layers of composite having CFCF condition with Six different orientations are considered, by keeping a/h ratio as constant which is equal to 0.04. In the case of (0/90/90/0/0/90/90/0) orientation gives the better results as compared with the other orientations. Similarly all others cases are summarizes in the table 8 and the figure no. shows the plot of 8 layers of laminated composite with CFCF conditions. But plate is affected less when Hole is at the center of the plate.

Table NO. 10 EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8  
LAYERS OF LAMINATED COMPOSITE WITH CFCF

	(0/90/90/0)s	(0/60/60/0)s	(0/45/45/0)s	(0/30/30/0)s	(0/15/30/45)s	(0/30/60/90)s
NO HOLES	33.09	25.57	19.569	15.521	14.74	19.783
HOLE AT CENTRE	34.915	31.201	29.588	28.944	28.75	30.412
HOLE AT EDGE	28.239	27.797	26.242	24.514	23.686	26.106
HOLE AT BOTH EDGES	34.033	31.377	29.144	27.762	26.626	29.609
HOLE AT SUPPORT	33.95	30.209	27.998	26.948	27.072	28.802
HOLE AT SUPPORT(CORNER)	28.272	25.205	23.883	22.816	22.404	24.318

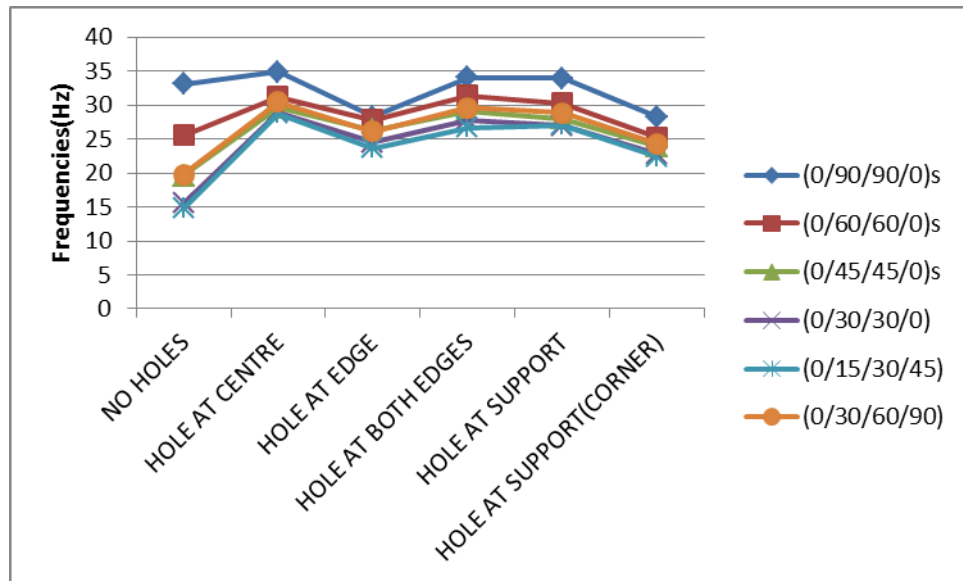


Figure 25- EFFECT OF FIBRE ORIENTATIONS FOR FREQUENCIES HAVING 8 LAYERS  
OF LAMINATED COMPOSITE WITH CFCF

## 6. CONCLUSION

### 6.1 INTRODUCTION

In the present work, the conventional finite element method using ANSYS is used to study the dynamic behavior of laminated composite plates with and without holes for the effects on the free vibration of plates. The numerical results are presented and discussed in above. The broad conclusions that can be made from the present study are summarized as follows:

The fundamental natural frequency of vibration decreases with increasing in  $a/h$  ratio.

1. When there is no hole in the plate with ply orientation of (0/30/30/0) shows highest frequency vibration when  $a/h$  ratio is equal to 50 for CFFF and CFCF boundary conditions.
2. When there is no hole in the plate with ply orientation of (0/30/60/90) shows highest frequency vibration when  $a/h$  ratio is equal to 200 for CFFF and CFCF boundary conditions.
3. When there is no hole in the plate with ply orientation of (0/15/30/45)s shows highest frequency vibration when  $a/h$  ratio is equal to 50 for CFFF and CFCF boundary conditions.
4. When there is no hole in the plate with ply orientation of (0/90/90/0)s shows highest frequency vibration when  $a/h$  ratio is equal to 200 for CFFF and CFCF boundary conditions.

In all other conditions for both  $a/h$  ratios 50 and 200, shows the highest frequencies as compared with others for 4 layers (0/90/90/0) and for 8 layers (0/90/90/0)s.

We conclude that the SHELL 99 element for meshing the carbon composite specimens yielded results with good accuracy. Hence, we recommend the use of SHELL 99 for Modal Analysis of carbon fiber composite specimens.

From the present studies, it is concluded that the best location for hole at both  $a/h$  ratios for cantilever, hole at the extreme edges similarly. Best location for hole at both  $a/h$  ratios for CFCF condition, hole at mid-section of plate.

## **6.2 FURTHER SCOPE**

Buckling analysis can be included

The present investigation can be extended to dynamic stability of laminated plates and shells subjected to hydrothermal condition

Material and geometry nonlinearity may be taken into account in the formulation for further extension of the dynamic stability of plates.

The effects of damping on instability regions of plates can be studied.

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